## Statistical Machine Translation

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## Language Technology II

Based on Kevin Knight's 1999 SS 2014

## A Statistical MT Tutorial Work Book

■ Introduction: the basic idea

- IBM models: the noisy channel, Model 3, EM ■ Phrase-Based SMT




## Today: Model $3 P(f \mid e)$ in full glory:

$$
\begin{aligned}
P(a, f \mid e)= & \binom{m-\varphi_{0}}{\varphi_{0}} \times p_{0}{ }^{\left(m-2 \varphi_{0}\right)} \times p_{1} \varphi_{0} \\
& \times \prod_{i=1}^{l} n\left(\varphi_{i} \mid e_{i}\right) \times \prod_{j=1}^{m} t\left(f_{j} \mid e_{a_{j}}\right) \\
& \times \prod_{j: a_{j} \neq 0}^{m} d\left(j \mid a_{j}, l, m\right) \times \prod_{i=0}^{l} \varphi_{i}!\times \frac{1}{\varphi_{0}!}
\end{aligned}
$$

Recall that

$$
P(f \mid e)=\sum_{a} P(a, f \mid e) \quad \text { and } \quad P(a \mid e, f)=\frac{P(a, f \mid e)}{\sum_{a} P(a, f \mid e)}
$$

## Translation Modelling

■ Remember that translating $f$ to $e$ we reason backwards
■ We observe $f$
$\square$ We want to know what $e$ is (most) likely to be uttered and likely to have been translated into $f$

$$
\hat{e}=\arg \max _{e} P(f \mid e) \times P(e)
$$

- Story: replace words in $e$ by $f$ (French) words and scramble them around
■ "What kind of a crackpot story is that?" (Kevin Knight, 1999)

■ IBM Model 3 -

## What happens in translation?

■ What happens in translation?
■ Actually a lot ....

- EN: Mary did not slap the green witch
- ES: Mary no daba una botefada a la bruja verde
- But from a purely external point of view
$\square$ Source words get replaced by target words
$\square$ Words in target are moved around ("reordered")
$\square$ Source and target need not be equally long ....
- So minimally that is what we need to model ...


## Model parameters

1. For each word $e_{i}$ in an English sentence $i=(1 \ldots l)$, we choose a fertility $\varphi_{i}$. The choice of fertility is dependent solely on the English word in question, nothing else.
2. For each word $e_{i}$, we generate $\varphi_{i}$ French words: $t(f \mid e)$. The choice of French word is dependent solely on the English word that generates it. It is not dependent on the English context around the English word. It is not dependent on other French words that have been generated from this or any other English word.
3. All those French words are permuted: $d\left(\pi_{f} \mid \pi_{e}, l, m\right)$. Each French word is assigned an absolute target "position slot." For example, one word may be assigned position 3, and another word may be assigned position 2 -- the latter word would then precede the former in the final French sentence. The choice of position for a French word is dependent solely on the absolute position of the English word that generates it.

Mary did not slap the green witch


Mary $\varnothing$ not slap slap slap the the green witch


Maria no daba una bofetada a la verde bruja


Maria no daba una bofetada a la bruja verde

## Model parameters

■ We would like to learn the Parameters for fertility, (word) translation and distortion from data

- The parameters look like this
$\square n(3 \mid$ slap $)$
$\square t$ (maison|house)
$\square d(5 \mid 2,4,6)$

■ And they have probabilities associated with them

■ One more twist: spurious words
■ E.g. function words can appear in target that do not have correspondences in source

- Pretend that every English sentence has NULL word in position 0 and can generate spurious words in target: $t(a \mid N U L L)$
■ Longer sentences are more likely to have more spurious words, therefore:
■ $N U L L$ doesn't have fertility distribution $n$ but a probability $p_{1}$ with which it can generate a spurious word after each properly generated word, how many decided by $\varphi_{0}$
- $p_{0}=1-p_{1}$ is probability of not tossing in spurious word


## NULL Mary did not slap the green witch

Mary $\varnothing$ not slap slap slap the green witch


Mary not slap slap slap NULL the green witch


Maria no daba una bofetada a la verde bruja


Maria no daba una bofetada a la bruja verde

1. For each English word $e_{i}$ indexed by $i=1,2, \ldots, l$ choose fertility $\varphi_{i}$ with probability $n\left(\varphi_{i} \mid e_{i}\right)$.
2. Choose the number $\varphi_{0}$ of "spurious" French words to be generated from $e_{0}=N U L L$, using probability $p_{1}$ and the sum of fertilities from step 1.
3. Let $m$ be the sum of fertilities for all words, including $N U L L$.
4. For each $i=1,2, \ldots, l$ and each $k=1,2, \ldots, \varphi_{i}$ choose a French word $\tau_{i, k}$ with probability $t\left(\tau_{i, k} \mid e_{i}\right)$.
5. For each each $i=1,2, \ldots, l$ and each $k=1,2, \ldots, \varphi_{i}$ choose target French position $\pi_{i, k}$ with probability $d\left(\pi_{i, k} \mid i, l, m\right)$.
6. For each $k=1,2, \ldots, \varphi_{0}$ choose a position $\pi_{0, k}$ from the $\varphi_{0}-k+1$ remaining vacant positions in $1,2, \ldots, m$ for a total probability of $1 / \varphi_{0}$ !.
7. Output the French sentence with words $\tau_{i, k}$ in positions $\pi_{i, k}(0 \leq i \leq$ $\left.l, 1 \leq k \leq \varphi_{i}\right)$.

## Model 3

Model 3 has four types of parameters

$$
n, t, p \text { and } d
$$

Need to think about two things:

■ How to get parameter values from data

■ Once we have those, how to compute $P(f \mid e)$ for any sentences $e$ and $f$

## Model 3 as String Rewriting

## NULL Mary did not slap the green witch

Mary $\varnothing$ not slap slap slap the green witch


Mary not slap slap slap NULL the green witch


Maria no daba una bofetada a la verde bruja


Maria no daba una bofetada a la bruja verde

## Model 3 as String Rewriting

NULL Mary did not slap the green witch
Mary $\varnothing$ not slap slap slap the green witch

Mary not slap slap slap NULL the green witch

Maria no daba una bofetada a la verde bruja

Maria no daba una bofetada a la bruja verde

## Model 3 as String Rewriting

NULL Mary did not slap the green witch
Mary $\varnothing$ not slap slap slap the green witch
Mary not slap slap slap NULL the green witch
Maria no daba una bofetada a la verde bruja
Maria no daba una bofetada a la bruja verde

- If we had a million English - French translations
-     + their step by step rewrites
- We could easily estimate parameter
- Use MLE: just count and divide


## Model 3 as String Rewriting

NULL Mary did not slap the green witch
Mary $\varnothing$ not slap slap slap the green witch
Mary not slap slap slap NULL the green witch
Maria no daba una bofetada a la verde bruja
Maria no daba una bofetada a la bruja verde

- If did occurred 15,000 times
- and did $\rightarrow \varnothing$ occurred 2000 times
- Then $n(0 \mid$ did $)=2 / 15$


## Model 3 as String Rewriting

## Exercise. <br> Take 10,000 English sentences and rewrite them into French, storing all intermediate strings. No, make that a million English sentences! Ha, ha, just kidding. Don't do this exercise.

Kevin Knight, A Statistical MT Tutorial Workbook, 1999, p. 14

## Word-for-Word Alignments

Our generative model in terms of string rewriting:

NULL And the program has been implemented
the program has been implemented implemented implemented

| Le programme a ete | mis | en | application |
| :--- | :--- | :--- | :--- | :--- |
| Le programme a ete | mis | en | application |

A simple data-structure that captures (most) of this: alignments


## Word-for-Word Alignments



Word alignments that correspond best to Model 3 :

■ Every French word connected to exactly one English word (incl. NULL)

■ So we never have 2 (or more) English generate one French

- $[2,3,4,5,6,6,6]$


## Word-for-Word Alignments and MLE Parameter Estimation: $n$ and $t$



If we had a million of these we could estimate Model 3 parameters (MLE):

$$
\begin{aligned}
& n(0 \mid \text { the })=\operatorname{count}(\text { the } \rightarrow \emptyset) / \operatorname{count}(\text { the }) \\
& t(\text { la } \mid \text { the })=\operatorname{count}(\text { the } \rightarrow \text { la }) / \text { count }(\text { the }) \\
& n(0 \mid \text { the })=c(\text { the } \rightarrow \emptyset) / c(\text { the }) \\
& t(\text { la } \mid \text { the })=c(\text { the } \rightarrow \text { la }) / c(\text { the }) \\
& n(0 \mid \text { the })=\#(\text { the } \rightarrow \emptyset) / \#(\text { the }) \\
& t(\text { la } \mid \text { the })=\#(\text { the } \rightarrow \text { la }) / \#(\text { the })
\end{aligned}
$$

# Word-for-Word Alignments and MLE Parameter Estimation: $d$ and $p$ 

- $d(5 \mid 2,4,6)$
- English word 2 in French position 5, where English sentence is 4 words long and French 6

■ How to estimate probability distribution over $d(j \mid 2,4,6)$ ?

$$
d(5 \mid 2,4,6)=\frac{\# d(5 \mid 2,4,6)}{\sum_{j=1}^{6} \# d(j \mid 2,4,6)}
$$

## Word-for-Word Alignments and MLE Parameter Estimation: d and p

- $p_{1}$ is probability for tossing in "spurious" NULL generated word after each properly generated word

■ In aligned data: $M$ words generated by NULL. Then $M$ spurious words will be generated in $N-M$ cases:

$$
p_{1}=\frac{M}{N-M}
$$

■ If we had large aligned data, we could estimate our parameters

- Unfortunately we don't have such data
- Bootstrapping

■ Learning with/from incomplete data: we have the translations but not the alignments $a$

## Word-for-Word Alignments and MLE Parameter Estimation:

■ If we had large aligned data, we could estimate our parameters
■ Unfortunately we don't have such data

■ If we had the parameters we could estimate alignments

- Unfortunately we don't have such data
- Bootstrapping
- Learning with/from incomplete data: we have the translations but not the alignments $a$


## A Chicken and Egg Problem

■ If we had alignments, we could estimate model parameters (such as translation probabilities, fertilities etc.)
■ If we had model parameters, we could estimate alignments

- We don't want to/can't spend a lot of money to manually align 100s of thousands (or millions) of sentences of bi-text
■ Need a way of estimating model parameters from incomplete data
- The thing we don't have is called a "hidden" variable (a "latent" variable, unobserved ...)
- In our case this is the alignment $a, a$ is a latent variable

■ Expectation Maximisation

## Expectation Maximisation (EM)

$\square$ Alignment $a, a$ is a latent variable

- Incomplete data

■ Learning from incomplete data

■ Up to know, we always have learned from complete data
■ Maximum Likelihood Estimation (MLE)
■ Maximises the likelihood of the data

■ Now parts of the data missing:
■ Expectation Maximisation (EM)

## Expectation Maximisation (EM)

From: Chuong B do \& Serafin Batzoglou, 2008
a Maximum likelihood


| Coin A | Coin B |  |
| :--- | :---: | :---: |
|  | $5 \mathrm{H}, 5 \mathrm{~T}$ |  |
| $9 \mathrm{H}, 1 \mathrm{~T}$ |  | $\hat{\theta}_{A}=\frac{24}{24+6}=0.80$ |
| $8 \mathrm{H}, 2 \mathrm{~T}$ |  |  |
|  | $4 \mathrm{H}, 6 \mathrm{~T}$ |  |
| $7 \mathrm{H}, 3 \mathrm{~T}$ |  |  |
| $24 \mathrm{H}, 6 \mathrm{~T}$ | $9 \mathrm{H}, 11 \mathrm{~T}$ |  |

## Expectation Maximisation (EM)



## Expectation Maximisation (EM)

- EM is a bit like magic ©

■ Kind of reduces incomplete data setting to complete data

- Converges
- But not perfect:
- Only local maximum
- A few other constraints
- But very common:
$\square$ e.g. Baum-Welsh for estimating HMMs
$\square$ Many others


## Translation model $P(f \mid e)$

■ In estimating IBM Model-3 $P(f \mid e)$ parameters the latent variable is the alignment $a$
■ Given no further information/knowledge what is our best guess about $a$ ?
■ Best here means least bias ...

- As starting point
- We have to assume that given a sentence pair a can align any word with any other word
■ That is many alignments ...


## Many alignments/All alignments

| $b c$ | $b c$ | $b c$ | $b c$ |
| :---: | :---: | :---: | :---: |
| $\downarrow \downarrow$ | $\swarrow$ | $\swarrow$ | $X$ |
| xy | xy | xy | xy |

To handle parameter estimation if you assume multiple/all alignments you could use fractional counts

$$
t(x \mid b)=\frac{\#(b \rightarrow x)}{\# b}
$$

## Many alignments/All alignments

| bc | bc | bc | bc |
| :---: | :---: | :---: | :---: |
| $\downarrow \downarrow$ | $\downarrow$ | $\downarrow$ | $\searrow$ |
| x y | xy | xy | xy |
| $w_{1}$ | $w_{2}$ | $w_{3}$ | $w_{4}$ |
| 0.3 | 0.2 | 0.4 | 0.1 |

- Alignments may also have weights $w$ associated with them
- Some more important than others ...
- These weights are then reflected in the counts for estimating parameters:

$$
n(1 \mid b)=\frac{0.3+0.1}{0.3+0.1+0.2+0.4}
$$

## Probability of alignment $a$

■ Weights are just one step away from probabilities

- Probability of alignment $a$ given $e$ and $f$ :

$$
P(a \mid e, f)=\frac{P(a, f \mid e)}{P(f \mid e)}
$$

- What makes one $a$ better than another?
- If e.g. many words aligned are likely translations of each other
■ i.e. have high $t$ parameter values


## Probability of alignment $a$

$$
P(a \mid e, f)=\frac{P(a, f \mid e)}{P(f \mid e)}
$$

■ Need to compute $P(a, f \mid e)$ and $P(f \mid e)$
■ $P(a, f \mid e) \ldots$ Model 3: the generative story gives you $f$ and $a$
■ In a way $a$ is a summary of the choices in Model 3

- $P(f \mid e) \ldots$ given an $e$, (many) alignments a may give you same $f$

$$
P(f \mid e)=\sum_{a} P(a, f \mid e)
$$

- Both $P(a, f \mid e)$ and $P(f \mid e)$ reduced to $P(a, f \mid e)$
- $P(a, f \mid e)$ is a product of a bunch of smaller probabilities (parameters)
- For each source word $e_{i}$ choosing fertility $n$, a translation $t$ and a target position $d$ :
$P(a, f \mid e)=\prod_{i=1}^{l} n\left(\varphi_{i} \mid e_{i}\right) \times \prod_{j=1}^{m} t\left(f_{j} \mid e_{a_{j}}\right) \times \prod_{j=1}^{m} d\left(j \mid a_{j}, l, m\right)$
- $l$ is the length of the English Sentence
- $m$ is the length of the French Sentence
- In case you forgot: we are translating "backwards" $t\left(f_{j} \mid e_{a_{j}}\right)$ because of Noisy Channel Model ...


## $P(a, f \mid e)$ a few Refinements

$$
P(a, f \mid e)=\prod_{i=1}^{l} n\left(\varphi_{i} \mid e_{i}\right) \times \prod_{j=1}^{m} t\left(f_{j} \mid e_{a_{j}}\right) \times \prod_{j=1}^{m} d\left(j \mid a_{j}, l, m\right)
$$

■ d should only apply to French words generated by real English words, and not by $N U L L: \prod_{j: a_{j} \neq 0}^{m} d\left(j \mid a_{j}, l, m\right)$
■ Need to include costs for $\varphi_{0}$ "spurious" NULL generated French words: there are $m-\varphi_{0}$ non-spurious French words, hence $\binom{m-\varphi_{0}}{\varphi_{0}}$ ways/positions of generating "spurious" words
■ What about the costs for this? For $\varphi_{0}$ "spurious" words: $p_{1} \varphi_{0}$ For the $\left(m-\varphi_{0}-\varphi_{0}\right)$ don't add spurious: $p_{0}{ }^{\left(m-2 \varphi_{0}\right)}$

## $P(a, f \mid e)$ a few Refinements

$$
P(a, f \mid e)=\prod_{i=1}^{l} n\left(\varphi_{i} \mid e_{i}\right) \times \prod_{j=1}^{m} t\left(f_{j} \mid e_{a_{j}}\right) \times \prod_{j=1}^{m} d\left(j \mid a_{j}, l, m\right)
$$

■ We have no costs for permuting spurious French words into their target positions: $d$ should only apply to French words generated by real English words, and not by NULL: $\prod_{j: a_{j} \neq 0}^{m} d\left(j \mid a_{j}, l, m\right)$
■ Once we have generated $\varphi_{0}$ spurious words we have $\varphi_{0}$ ! ways of permuting them, each of them with a probability of $\frac{1}{\varphi_{0}!}$

## $P(a, f \mid e)$ a few Refinements

$$
P(a, f \mid e)=\prod_{i=1}^{l} n\left(\varphi_{i} \mid e_{i}\right) \times \prod_{j=1}^{m} t\left(f_{j} \mid e_{a_{j}}\right) \times \prod_{j=1}^{m} d\left(j \mid a_{j}, l, m\right)
$$

- There is a final problem: the alignment loses a bit of info about how $e$ can get turned into $f$ by the generative process:
- If English $x$ is connected to both French $z$ and $y, a$ doesn't tell us whether they were generated in that order, or as $y$ followed by $z$ and then permuted $\ldots$ similarly for when $x$ is connected with three (or more) French words
- We add a factor

$$
\prod_{i=0}^{l} \varphi_{i}!
$$

## $P(a, f \mid e)$ in full glory:

$$
\begin{aligned}
P(a, f \mid e)= & \binom{m-\varphi_{0}}{\varphi_{0}} \times p_{0}{ }^{\left(m-2 \varphi_{0}\right)} \times p_{1} \varphi_{0} \\
& \times \prod_{i=1}^{l} n\left(\varphi_{i} \mid e_{i}\right) \times \prod_{j=1}^{m} t\left(f_{j} \mid e_{a_{j}}\right) \\
& \times \prod_{j: a_{j} \neq 0}^{m} d\left(j \mid a_{j}, l, m\right) \times \prod_{i=0}^{l} \varphi_{i}!\times \frac{1}{\varphi_{0}!}
\end{aligned}
$$

$$
\begin{aligned}
P(a, f \mid e)= & \binom{m-\varphi_{0}}{\varphi_{0}} \times p_{0}{ }^{\left(m-2 \varphi_{0}\right)} \times p_{1} \varphi_{0} \\
& \times \prod_{i=1}^{l} n\left(\varphi_{i} \mid e_{i}\right) \times \prod_{j=1}^{m} t\left(f_{j} \mid e_{a_{j}}\right) \\
& \times \prod_{j: a_{j} \neq 0}^{m} d\left(j \mid a_{j}, l, m\right) \times \prod_{i=0}^{l} \varphi_{i}!\times \frac{1}{\varphi_{0}!}
\end{aligned}
$$

Recall that

$$
P(f \mid e)=\sum_{a} P(a, f \mid e) \quad \text { and } \quad P(a \mid e, f)=\frac{P(a, f \mid e)}{\sum_{a} P(a, f \mid e)}
$$

## Chicken and Eggs

- If we have parameters we can compute alignments

■ If we have alignments we can compute parameters

$$
P(f \mid e)=\sum_{a} P(a, f \mid e) \quad \text { and } \quad P(a \mid e, f)=\frac{P(a, f \mid e)}{\sum_{a} P(a, f \mid e)}
$$

## Expectation Maximisation to the Rescue

■ Start with uniform parameter values
$\square$ Let French vocab $=40,000$ words
$\square$ Then $t(f \mid e)=\frac{1}{40,000}$ for each $e$
$\square d(4 \mid 4,10,10)=\frac{1}{10}$
$\square$ Pick random value for $p_{1}$, say 0.15

- With this we can compute alignment probabilities a for each pair of sentences
■ Collect fractional counts, normalise => better parameter values => better alignment probabilities => revised parameter values => and so on ...


## A two Sentence Example

| $b c$ | $b c$ | $b$ |
| :---: | :---: | :---: |
| $\downarrow \downarrow$ | $x$ | $\downarrow$ |
| $x y$ | $x y$ | $y$ |

- Simplifications: no $N U L L, \varphi=1$ (always), no $d$
- Only $t$ impacts on $a$
- So $P(a, f \mid e)$ reduces to

$$
P(a, f \mid e)=\prod_{j=1}^{m} t\left(f_{j} \mid e_{a_{j}}\right)
$$

■ ~ IBM Model 1 (except IBM Model 1 has NULL )

## A two Sentence Example



## Here goes EM ... (Round 1)

■ Step 1: uniform parameters

$$
t(x \mid b)=t(y \mid b)=t(x \mid c)=t(y \mid c)=\frac{1}{2}
$$

■ Step 2: compute $P(a, f \mid e)$ for all alignments

$$
P(a, f \mid e)=\prod_{j=1}^{m} t\left(f_{j} \mid e_{a_{j}}\right)
$$

$$
P\left(a_{1}, f \mid e\right)=\frac{1}{2} \times \frac{1}{2}=\frac{1}{4} \quad P\left(a_{2}, f \mid e\right)=\frac{1}{2} \times \frac{1}{2}=\frac{1}{4} \quad P\left(a_{3}, f \mid e\right)=\frac{1}{2}
$$

- Step 3: normalise $P(a, f \mid e)$ to get $P(a \mid f, e)$

$$
\begin{aligned}
& P \text { 3: normalise } P(a, f \mid e) \text { to get } P(a \mid f, e) \\
& P\left(a_{1} \mid f, e\right)=\frac{\frac{1}{4}}{\frac{2}{4}}=\frac{1}{2} \quad P\left(a_{2} \mid f, e\right)=\frac{\frac{1}{4}}{\frac{2}{4}}=\frac{1}{2} \quad P\left(a_{3} \mid f, e\right)=\frac{\frac{1}{2}}{\frac{1}{2}}=1
\end{aligned}
$$

$$
P(a \mid e, f)=\frac{P(a, f \mid e)}{\sum_{a} P(a, f \mid e)}
$$

## A two Sentence Example

| bc | bc | b |
| :---: | :--- | :--- |
| $\downarrow \downarrow a_{1}$ | $\mathrm{X}_{\mathrm{a}} a_{2}$ | $\downarrow a_{3}$ |
| x y | xy | y |

## Here goes EM ... (Round 1)

■ Step 4: collect fractional counts (weighted by alignment probabilities from Step 3)

$$
\# t(x \mid b)=\frac{1}{2} \quad \# t(y \mid b)=\frac{1}{2}+1=\frac{3}{2} \quad \# t(x \mid c)=\frac{1}{2} \quad \# t(y \mid c)=\frac{1}{2}
$$

■ Step 5: normalise fractional counts to get revised parameters $t$

$$
t(x \mid b)=\frac{\frac{1}{2}}{\frac{4}{2}}=\frac{1}{4} \quad t(y \mid b)=\frac{\frac{3}{2}}{\frac{4}{2}}=\frac{3}{4} \quad t(x \mid c)=\frac{1}{2} / 1=\frac{1}{2} \quad t(y \mid c)=\frac{1}{2} / 1=\frac{1}{2}
$$

Normalised by sum of factional counts from Step 4 ...

## A two Sentence Example

$$
\begin{array}{lll}
\text { b c } & \text { b c } & \text { b } \\
\downarrow \downarrow a_{1} \bigotimes_{a} & \downarrow a_{3} & \mathrm{y}
\end{array}
$$

## Here goes EM ... (Round 1)

■ Step 4: collect fractional counts (weighted by alignment probabilities from Step 3)

$$
\# t(x \mid b)=\frac{1}{2} \quad \# t(y \mid b)=\frac{1}{2}+1=\frac{3}{2} \quad \# t(x \mid c)=\frac{1}{2} \quad \# t(y \mid c)=\frac{1}{2}
$$

■ Step 5: normalise fractional counts to get revised parameters $t$

$$
\begin{aligned}
& t(x \mid b)=\frac{\frac{1}{2}}{\frac{4}{2}}=\frac{1}{4} \quad t(y \mid b)=\frac{\frac{3}{2}}{\frac{4}{2}}=\frac{3}{4} \quad t(x \mid c)=\frac{1}{2} / 1=\frac{1}{2} \quad t(y \mid c)=\frac{1}{2} / 1=\frac{1}{2}
\end{aligned}
$$

## A two Sentence Example

| bc | bc | b |
| :---: | :---: | :---: |
| $\downarrow \downarrow a_{1}$ | $\searrow a_{2}$ | $\downarrow a_{3}$ |
| x y | xy | y |

## Here goes EM ... (Round 2)

- Repeat Step 2: compute $P(a, f \mid e)$ for all alignments

$$
P(a, f \mid e)=\prod_{j=1}^{m} t\left(f_{j} \mid e_{a j}\right)
$$

$$
P\left(a_{1}, f \mid e\right)=\frac{1}{4} \times \frac{1}{2}=\frac{1}{8} \quad P\left(a_{2}, f \mid e\right)=\frac{3}{4} \times \frac{1}{2}=\frac{3}{8} \quad P\left(a_{3}, f \mid e\right)=\frac{3}{4}
$$

- Repeat Step 3: normalise $P(a, f \mid e)$ to get $P(a \mid f, e)$

$$
P(a \mid e, f)=\frac{P(a, f \mid e)}{\sum_{a} P(a, f \mid e)}
$$

$$
P\left(a_{1} \mid f, e\right)=\frac{\frac{1}{8}}{\frac{4}{4}}=\frac{1}{4} \quad P\left(a_{2} \mid f, e\right)=\frac{\frac{3}{8}}{\frac{4}{8}}=\frac{3}{4} \quad P\left(a_{3} \mid f, e\right)=\frac{\frac{3}{4}}{\frac{3}{4}}=1
$$

## A two Sentence Example



## Here goes EM ... (Round 2)

■ Repeat Step 4: collect fractional counts (weighted by alignment probabilities from Repeat Step 3):

$$
\# t(x \mid b)=\frac{1}{4} \quad \# t(y \mid b)=\frac{3}{4}+1=\frac{7}{4} \quad \# t(x \mid c)=\frac{3}{4} \quad \# t(y \mid c)=\frac{1}{4}
$$

■ Repeat Step 5: normalise fractional counts to get revised parameters $t$

$$
t(x \mid b)=\frac{\frac{1}{4}}{\frac{8}{4}}=\frac{1}{8} \quad t(y \mid b)=\frac{\frac{7}{4}}{\frac{8}{4}}=\frac{7}{8} \quad t(x \mid c)=\frac{3}{4} / 1=\frac{3}{4} \quad t(y \mid c)=\frac{1}{4} / 1=\frac{1}{4}
$$

## A two Sentence Example

| bc | bc | b |
| :---: | :--- | :--- |
| $\downarrow \downarrow a_{1}$ | $\searrow a_{2}$ | $\downarrow a_{3}$ |
| x y | xy | y |

## Here goes EM ... (Round 2)

■ Repeat Step 4: collect fractional counts (weighted by alignment probabilities from Step 3):

$$
\# t(x \mid b)=\frac{1}{4} \quad \# t(y \mid b)=\frac{3}{4}+1=\frac{7}{4} \quad \# t(x \mid c)=\frac{3}{4} \quad \# t(y \mid c)=\frac{1}{4}
$$

■ Repeat Step 5: normalise fractional counts to get revised parameters $t$

$$
t(x \mid b)=\frac{\frac{1}{4}}{\frac{8}{4}}=\frac{1}{4} \quad t(y \mid b)=\frac{\frac{7}{4}}{\frac{8}{4}}=\frac{7}{8} \quad t(x \mid c)=\frac{3}{4} / 1=\frac{3}{4} \quad t(y \mid c)=\frac{1}{4} / 1=\frac{1}{4}
$$

$$
\begin{array}{lll}
\mathrm{b} \mathrm{c} & \mathrm{~b} \mathrm{c} & \mathrm{~b} \\
\downarrow \downarrow a_{1} & \mathbf{K} a_{2} & \downarrow a_{3} \\
\mathrm{xy} & \mathrm{xy} & \mathrm{y}
\end{array}
$$

## A two Sentence Example

| bc | bc | b |
| :---: | :--- | :--- |
| $\downarrow \downarrow a_{1}$ | $\mathrm{X}_{\mathrm{a}}$ | $\downarrow a_{3}$ |
| x y | xy | y |

## Here goes EM ...

■ Repeating Steps 2-5 many times:

$$
t(x \mid b)=0.0001 \quad t(y \mid b)=0.9999 \quad t(x \mid c)=0.9999 \quad t(y \mid c)=0.0001
$$



